Physics 101
Ch.1. Units and Dimenstions
Lecture 1

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Physics

- **Fundamental Science**
  - **Concerned with the fundamental principles of the Universe**
  - **Foundation of other physical sciences**
  - **Has simplicity of fundamental concepts**
- **Divided into five major areas**
  - **Classical Mechanics**
  - **Relativity**
  - **Thermodynamics**
  - **Electromagnetism**
  - **Optics**
  - **Quantum Mechanics**

Measurements

- **Used to describe natural phenomena**
- **Needs defined standards**
- **Characteristics of standards for measurements**
  - **Readily accessible**
  - **Possess some property that can be measured reliably**
  - **Must yield the same results when used by anyone anywhere**
  - **Cannot change with time**

Physics and the Laws of Universe

Physics: the study of the relations between physical quantities, expressed as fundamental laws of nature.

- These laws can be expressed as mathematical **equations**. **(e.g., F = m a)**
- Most physical quantities have **units**, which must match on both sides of an equation.
- Much **complexity** can arise from even relatively simple physical laws.

Units

With a few exceptions, all physical quantities have units. Examples:

Mass	-	kilograms (kg)
Speed	-	meters per second (m/s)
Pressure	-	pascals (P)
Energy	-	joules (J)
Electric Potential	-	volts (V)

Rather surprisingly, the units of almost *all* physical quantities can be expressed as combinations of *only* the units for *mass*, *length*, and *time*, i.e., kilograms, meters, and seconds. A few physical quantities (e.g., index of refraction) are pure numbers that have no associated units.

Standard International Units

SI – Systéme International :

Agreed to in 1960 by an international committee

Standard International (SI) Units (also known as MKS)

Dimension	SI (mks) Unit	Definition
Length	meters (m)	this latest definition establishes that the speed of light in vacume is precisely 1/299,792,458) meter/second
Mass	kilogram (kg)	Mass of a specific platinum-iridium allow cylinder kept by Intl. Bureau of Weights and Measures at Sèvres, France
Time	seconds (s)	9,192,631,700 oscillations of cesium atom

Units for almost all other physical quantities can be constructed from mass, length, and time, so these are the fundamental units.

Fundamental Quantities and Their Units

Quantity	SI Unit
Length	meter
Mass	kilogram
Time	second
Temperature	Kelvin
Electric Current	Ampere
Luminous Intensity	Candela
Amount of Substance	mole

Dimensional Analysis

- Technique to check the correctness of an equation or to assist in deriving an equation
- Dimensions (length, mass, time, combinations) can be treated as algebraic quantities
  - add, subtract, multiply, divide
- Both sides of equation must have the same dimensions
- Any relationship can be correct only if the dimensions on both sides of the equation are the same
- Cannot give numerical factors: this is its limitation

Dimensional Analysis

Will also use derived quantities. These are other quantities that can be expressed in terms of the basic quantities Examples.

Table 1-2 Dimensions of Physical Quantities

Quantity	Symbol	Dimension
Area	A	L^2
Volume	V	L^3
Speed	v	L/T
Acceleration	a	L/T^2
Force	F	ML/T^2
Pressure (F/A)	p	M/LT^2
Density (M/V)	ρ	M/L^3
Energy	E	ML^2/T^2
Power (E/T)	P	ML^2/T^3

Example:
Area is the product of two lengths
Area is a derived quantity

Length
is the fundamental quantity

Dimensional Analysis

Any valid physical equation must be dimensionally consistent – each side must have the *same* dimensions.

From the Table:

Distance = velocity × time

Velocity = acceleration × time

Energy = mass × (velocity)^2

Dimensional Analysis (2)

Example:

The period P (T) of a swinging pendulum depends only on the length of the pendulum d (L) and the acceleration of gravity g (L/T^2).

Which of the following formulas for P *could* be correct ?

(a) $P = 2\pi\,(dg)^2$ **(b)** $P = 2\pi \dfrac{d}{g}$ **(c)** $P = 2\pi \sqrt{\dfrac{d}{g}}$

Dimensional Analysis (3)

Remember that P is in units of time (T), d is length (L) and g is acceleration (L/T^2).

The both sides must have the *same* units

Try equation (a). Try equation (b). Try equation (c).

$$\cancel{\left(L\cdot\frac{L}{T^2}\right)^2=\frac{L^4}{T^4}\neq T} \qquad \cancel{\frac{L}{\frac{L}{T^2}}=T^2\neq T} \qquad \sqrt{\frac{L}{\frac{L}{T^2}}}=\sqrt{T^2}=T$$

(a) $\boxed{P=2\pi(dg)^2}$ **(b)** $\boxed{P=2\pi\frac{d}{g}}$ **(c)** $\boxed{P=2\pi\sqrt{\frac{d}{g}}}$

Dimensional Analysis

- This is a very important tool to check your work
  - It's also very easy!

- **Example:**

  Doing a problem you get the answer for distance

  $d = v\, t^2$ (velocity x time^2)

  Quantity on left side = L

  Quantity on right side = L / T x T^2 = L x T

- Left units and right units don't match, so answer must be wrong !!

Lecture 1, *ACT 2*

- There is a famous Einstein's equation connecting energy and mass (relativistic). Using dimensional analysis find which is

(a) $E = mc$ **(b)** $E = mc^2$ **(c)** $E = mc^3$

- the correct form of this equation :

- **Note :**

← c is speed of light (L/T)

←E is energy ($M\,L^2 / T^2$)

Solution -> (b)

Dimensional Analysis

Checking equations with dimensional analysis:

$$x_f - x_i = v_i t + \frac{1}{2} a t^2$$

L

(L/T)T=L

$(L/T^2)T^2=L$

- **Each term must have same dimension**
- **Two variables can not be added if dimensions are different**
- **Multiplying variables is always fine**
- **Numbers (e.g. 1/2 or p) are dimensionless**

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EX.

Check the equation for dimensional consistency:

$$mgh = \frac{mc^2}{\sqrt{1-(v/c)^2}} - mc^2$$

Here, *m* is a mass, g is an acceleration, *c* is a velocity, *h* is a length

Example

Consider the equation:

$$m\frac{v^2}{r} = G\frac{Mm}{r^2}$$

Where *m* and *M* are masses, *r* is a radius and *v* is a velocity.
What are the dimensions of *G* ?

$L^3/(MT^2)$

Example

Given "x" has dimensions of distance, "u" has dimensions of velocity, "m" has dimensions of mass and "g" has dimensions of acceleration.

Is this equation dimensionally valid?

$$x = \frac{(4/3)ut}{1-(2gt^2/x)}$$

Yes

Is this equation dimensionally valid?

$$x = \frac{vt}{1-mgt^2}$$

No

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Dimensional Analysis to Determine a Power Law

- Determine powers in a proportionality
- Example: find the exponents in the expression $x \propto a^m t^n$
- You must have lengths on both sides **X = L**
- Acceleration has dimensions of **a =** L/T^2
- Time has dimensions of **t = T**
- Analysis gives $x \propto at^2$

$$L = L^m T^{-2m} T^n$$

1= m, -2m+n =0,

n = 2

$$X = a\,t^2$$

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Example 1: Show that the expression v= a t is dimensionally correct, where v represents speed, a acceleration, and t an instant of time.

$$LHS = [v] = \frac{L}{T}$$

$$RHS = [a\,t] = \frac{L}{T^2}T = \frac{L}{T}$$

It is clear that LHS=RHS. Therefore, the expression is dimensionally correct

US Customary System

- Still used in the US, but text will use SI

Quantity	Unit
Length	foot
Mass	slug
Time	second

Prefixes

- Prefixes correspond to powers of 10
- Each prefix has a specific name
- Each prefix has a specific abbreviation

- بادئة مقابله إلى قوى من 10
- لكلّ بادئة لها اسم خاصّة
- كلّ بادئة لها اختصارات خاصّة

Prefixes, cont.

- The prefixes can be used with any basic units
- They are multipliers of the basic unit
- Examples:
  - 1 mm = 10^{-3} m
  - 1 mg = 10^{-3} g

TABLE 1.4

Prefixes for Powers of Ten

Power	Prefix	Abbreviation	Power	Prefix	Abbreviation
10^{-24}	yocto	y	10^{3}	kilo	k
10^{-21}	zepto	z	10^{6}	mega	M
10^{-18}	atto	a	10^{9}	giga	G
10^{-15}	femto	f	10^{12}	tera	T
10^{-12}	pico	p	10^{15}	peta	P
10^{-9}	nano	n	10^{18}	exa	E
10^{-6}	micro	μ	10^{21}	zetta	Z
10^{-3}	milli	m	10^{24}	yotta	Y
10^{-2}	centi	c			
10^{-1}	deci	d			

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NOTE THAT

Basic Quantities	The dimension of	length	$[L]$
	The dimension of	time	$[T]$
	The dimension of	mass	$[m]$
	The dimension of	angle	
	The dimension of	temperature	$[\theta]$
	The dimension of	current	$[I]$
Derived Quantities	The dimension of	velocity	$[lt^{-1}]$
	The dimension of	acceleration	$[lt^{-2}]$
	The dimension of	force	$[mlt^{-2}]$
	The dimension of	frequency	$[t^{-1}]$
	The dimension of	density	$[ml^{-3}]$
	The dimension of	elastic modulus	$[m\,l^{-1}t^{-2}]$
	The dimension of	linear mass density	$[m\,l^{-1}]$
	The dimension of	pressure	$[ml^{-1}t^{-2}]$
	The dimension of	energy	$[ml^{2}t^{-2}]$
	The dimension of	electric charge	$[IT]$
	The dimension of	electric field	$[MLT^{-3}I^{-1}]$
	The dimension of	magnetic field	$[MT^{-2}I^{-1}]$

Conversion of Units

- When units are not consistent, you may need to convert to appropriate ones
- See Appendix A for an extensive list of conversion factors
- Units can be treated like algebraic quantities that can cancel each other out

Appendix A

1 ft = 12 in

1 in = 2.54 cm

1 m = 3.281 ft

1 mile = 1609 m

1 Lit = 10^3 cm^3

1 gallon = 3.79 Lit

1 N/m^2 = $1.451*10^{-4}$ Ib/in^2

1 Ib = 4.448

Conversion

- Always include units for every quantity, you can carry the units through the entire calculation
- Multiply original value by a ratio equal to one
- Example

$$15.0\,in = ?\,cm$$

$$15.0\,in\left(\frac{2.54\,cm}{1\,\cancel{in}}\right) = 38.1\,cm$$

  - Note the value inside the parentheses is equal to 1 since 1 in. is defined as 2.54 cm